Graduation of mortality

Abstract
Graduation of mortality is a complex activity, which has had and still has the interest of many demographers and actuaries. Its importance for insurance offices and pension funds is obvious and - may be even more - for scientific research as well.
Several methods of graduation have been developed. Here, we focus on graduation by a mathematical formula, i.e. Makeham's Law (1879).

We have Makeham's graduation formula:

$$l_x = k.s^x.g^{c^x}$$

obtained by integration from the formula for the force of mortality

$$\mu_x = A + Bc^x$$

which was derived from Gompertz formula

$$\mu_x = Bc^x$$

reflecting only mortality by deterioration.
Makeham added a constant A to it, bringing in a factor of chance.

By King-Hardy's system of equations:

$$\ln \frac{l_{u+h}}{l_u} = \sum_{i=0}^{h-1} \ln p_{u+1} = hA + Bc^u.\frac{c^{h-1}}{c-1} = H_j$$

for $u = x, x+h, x+2h$ and $j = 1, 2, 3$

we may find:

$$c^h = \frac{H_3 - H_2}{H_2 - H_1}$$

$$B = \frac{(H_2 - H_1)(c-1)}{c^x(c^h-1)^2}$$

$$A = \frac{1}{h}\left[H_1 - \frac{bc^x(c^h-1)}{c-1}\right]$$

References: Benjamin, B. and Pollard J.H.
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